MATHEMATICS -FORM 2

Number Theory

This chapter will teach you how to:

- define natural numbers, whole numbers, integers, rational and irrational numbers, and real numbers.
- add, subtract, multiply and divide.
- use the identity and inverse for addition and multiplication; and multiply and divide by zero.
- define and use the law of closure, commutative law, associative law and distributive law.
- solve problems dealing with the powers of numbers; and use a defined arithmetic operation.
- obtain the factors, prime factors and multiples of a number; and determine the L.C.M. and H.C.F. of a set of numbers.
- define the sets of square numbers, rectangular numbers, prime numbers, composite numbers, even numbers and odd numbers.
- define a sequence and determine a term in the sequence.
- understand and use the binary system, quinary system, octal system and denary system.

Set of Natural Numbers

The *set of natural numbers* is another name given to the *set of counting numbers* and it is represented by the *symbol N*.

The *set of natural numbers*, $N = \{1, 2, 3, \ldots\}$.

Set of Whole Numbers

The *set of whole numbers* is the *set of natural numbers* or *counting numbers* and *zero*. It is represented by the *symbol W*.

It should be obvious from the previous statements that *zero* is *not* a *natural number*. *Zero* is represented by the *symbol* 0 (*nought*).

The *set of whole numbers*, $W = \{0, 1, 2, 3, \ldots\}$.

Set of Integers

The *set of integers* can be accepted as the *set of negative* and *positive natural numbers* and *zero*.

Alternatively, the *set of integers* can be regarded as the *set of negative* and *positive whole numbers* including *zero*. Note, however, that *zero* is *neither positive nor negative*. That is, $\pm 0 = 0$.

The *set of integers* is represented by the *symbol Z*.

The *set of integers*, $Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.

Set of Rational Numbers

The *set of rational numbers* is really the *set of numbers* that *can* be written as *fractions*. It is the *set of negative* and *positive fractions, including zero*.

For example: $-\frac{2}{3}$, $-\frac{1}{2}$, $\frac{3}{5}$ and $\frac{8}{9}$.

A *rational number* can always be *written* as a *decimal*, whether *terminating* or *recurring*.

For example: 0.8, 0.65, 0.3 and 0.6.

It should be obvious from the statements above that the *set of rational numbers contains* the *set of integers*, since all *whole numbers* can be *written* with 1 as their *denominator*.

For example: $-5 = \frac{-5}{1}$, $6 = \frac{6}{1}$ and $0 = \frac{0}{1}$.

The *set of rational numbers* is represented by the *symbol Q*.

The *set of rational numbers*,

$Q = \left\{\frac{n}{d} : n \in Z, d \in Z, d \neq 0\text{, and, } n \text{ and } d \text{ have no common factor}\right\}$,

where n = the *numerator*,
d = the *denominator*

and $\frac{n}{d}$ = a *fraction* in its *simplest terms*.

From the statements above, it can be seen that:

(i) the *set of rational numbers contains* the *set of integers*;

(ii) the *set of integers contains* the *set of whole numbers*; and

(iii) the *set of whole numbers contains* the *set of natural numbers*.

So we can write:

$Q \supset Z \supset W \supset N$

or

$N \subset W \subset Z \subset Q$.

So we have the following Venn diagram representing the information stated previously.

Fig. 2.1 Venn diagram

Set of Irrational Numbers

The *set of irrational numbers* is the *set of numbers* that *cannot* be written as *fractions*. For example:

$-\sqrt{3}$, $\sqrt{7}$, $-\sqrt{\frac{5}{4}}$ and $\sqrt{\frac{2}{9}}$.

Further, when *irrational numbers* are *written* as *decimals* they *do not terminate* or *recur*.

For example:

$\pi = 3.141\,5927\ldots$ (correct to 7 decimal places) and

$\sqrt{3} = 1.732\,0508\ldots$ (correct to 7 decimal places).

The *set of irrational numbers* is represented by either the *symbol Q'* or the *symbol I*.

The *set of irrational numbers*,

$Q' \neq \left\{\frac{n}{d} : n \in Z, d \in Z, d \neq 0\text{, and, } n \text{ and } d \text{ have no common factor}\right\}'$.

Or the *set of irrational numbers*,

$I = \left\{\frac{n}{d} : n \in Z, d \in Z, d \neq 0\text{, and, } n \text{ and } d \text{ have no common factor}\right\}'$.

Set of Real Numbers

The *set of real numbers* is the *union* of the *set of rational numbers* and the *set of irrational numbers*, and it is represented by the *symbol R*.

Thus $R = Q \cup Q' = Q \cup I$, when $U = R$.

So we have the following Venn diagram representing the *set of real numbers*.

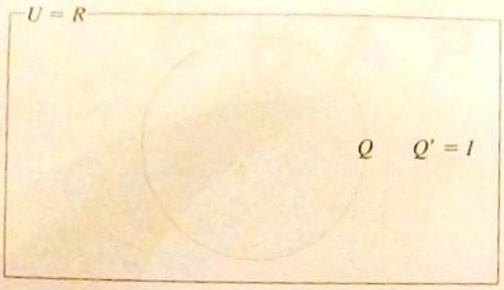

Fig. 2.2 Venn diagram

Alternatively, we have the more detailed Venn diagram representing the *set of real numbers*.

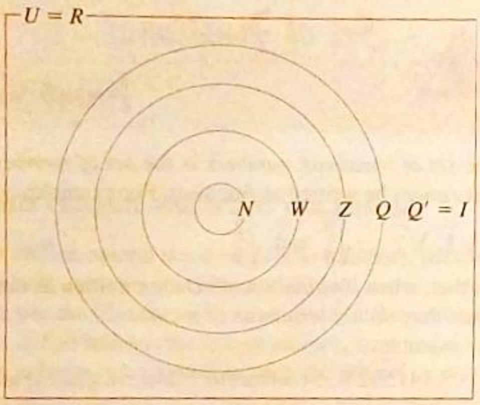

Fig. 2.3 Venn diagram

Thus $N \subset W \subset Z \subset Q \subset R$
and $Q' \subset R$.
So $R = Q \cup Q' = Q \cup I$.

Basic Arithmetic Operations

The *four basic arithmetic operations* are:

(1) *Addition*
(2) *Subtraction*
(3) *Multiplication*
(4) *Division*

Thus:

(1) To *add* means to calculate a *sum*.
For example:
(a) Add the numbers 4 and 9.
The *sum* of the numbers $= 4 + 9$
$= 13$.

(2) To *subtract* means to *take away* or to calculate a *difference*.
For example:
(b) Subtract the number 4 from the number 9.
The *difference* of the numbers $= 9 - 4$
$= 5$.

(3) To *multiply* means to calculate a *product*.
For example:
(c) Multiply the number 3 and 5.
The *product* of the numbers $= 3 \times 5$
$= 15$.

(4) To *divide* means to calculate a *quotient*.
For example:
(d) Divide the number 8 by the number 2.
The *quotient* of the numbers $= 8 \div 2$
$= \frac{8}{2}$
$= 4$.

Some Meanings of Zero

Some *meanings* of *zero* are:

(a) *Zero* is used to *indicate* an *empty place value* in any number with *more* than *one digit*. For example: 74035 indicates that there are *zero hundreds* in the number seventy-four thousand and thirty-five.

(b) *Zero* is also the *number of elements* in the *empty* or *null set*. That is $n(\varnothing) = 0$.

(c) *Zero* is used to *represent* the *mid-point* on the *number line* between -1 and 1, -2 and 2, -3 and 3, et cetera.

This *fact* can be seen *illustrated* below.

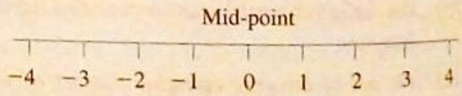

Fig. 2.4 Number line

(d) *Zero* can also be seen as the *identity* for the *addition of numbers*.

That is:
$4 + 0 = 4$
$0 + 5 = 5$
$-4 + 0 = -4$
$0 + (-5) = -5$

Identity for Addition

The *identity* for an *operation* leaves the *original number unchanged* under the operation.

If *zero* is *added* to any *number*, then the *sum* is the *original number*.

Thus:

$$4 + 0 = 4$$
$$0 + 3 = 3$$
$$-4 + 0 = -4$$
$$0 + (-3) = -3$$

We say that *zero* is the *identity* for the *addition of numbers*.

Identity for Multiplication

If any *number* is *multiplied* by 1, then the *product* is the *original number*.

Thus:

$$8 \times 1 = 8$$
$$1 \times 9 = 9$$
$$-8 \times 1 = -8$$
$$1 \times (-9) = -9$$

We say that 1 is the *identity* for the *multiplication of numbers*.

Inverse for Numbers Under Addition

The *inverse of a number* for a given *operation combines* with the *number* under the *operation* to give the *identity*.

Thus:

The *inverse of* 5 *under addition* is -5,
since $5 + (-5) = 0$ (*identity*).
The *inverse of* -3 *under addition* is 3,
since $-3 + 3 = 0$ (*identity*).

Inverse for Numbers Under Multiplication

The *definition* for the *inverse of a number* was stated above.

Thus:

The *inverse of* 6 *under multiplication* is $\frac{1}{6}$,
since $6 \times \frac{1}{6} = 1$ (*identity*).

The *inverse of* -7 *under multiplication* is $-\frac{1}{7}$,
since $-7 \times \left(-\frac{1}{7}\right) = 1$ (*identity*).

Multiplication by Zero

If any *number* is *multiplied by zero*, then the *product* is always *zero*.

Thus:

$$8 \times 0 = 0$$
$$0 \times 7 = 0$$
$$-3 \times 0 = 0$$
$$0 \times (-1) = 0$$

Division by Zero

If any *number* is *divided by zero*, then we say that the *result* is *infinity*.

Thus:

$$\frac{3}{0} = +\infty$$
$$\frac{-4}{0} = -\infty$$

Sometimes it is *easier* to say that *division by zero* is a *meaningless operation*.

However, the *quotient of zero divided* by any *number other than zero* is always *zero*.

Thus: $\frac{0}{1} = \frac{0}{5} = \frac{0}{-3} = \frac{0}{-4} = 0$

Law of Closure

The *law of closure* states that a *set of numbers* is *closed* under an *operation*, if when the operation is *performed* on any *two members of the set*, then the *result* is a *member of the set*.

Thus:

(a) $6 + 5 = 11$

(b) $3 \times 4 = 12$